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Voltage-Controlled Oscillator with Time-Delay Feedback

J. B. L. Rao and W. M. Waters

Search Radar Branch
Radar Division

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A voltage-controlled oscillator with time-delay feedback is analyzed. The results indicate that by properly choosing system parameters, both low noise from the degeneration and frequency agility from the periodic response can be obtained in one source. The conditions for system stability and the amount of noise degeneration are expressed in terms of system parameters. Effects of different parameters on system stability and noise degeneration are discussed in detail. It is shown that the open-loop gain should be chosen as large as possible for good noise degeneration, and the amplifier-filter bandwidth should be chosen to meet the stability condition.

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CONTENTS

VOLTAGE-CONTROLLED OSCILLATOR WITH TIME-DELAY FEEDBACK

INTRODUCTION

A voltage-controlled oscillator (VCO) with time-delay feedback has been designed as a frequency-agile, noise-degenerated radar frequency source. This frequency source uses a solid state VCO with a noise-degenerated negative feedback loop with an interferometer as a frequency discriminator. Because of the periodicity of responses of the interferometer, the oscillator operation is stable at many frequencies across the band of interest. For this reason, both low noise from the degeneration and frequency agility from the periodic response are obtained in one source. This report analyzes the steady state and transient responses of the frequency source. The steady state solution will provide information necessary in choosing the loop parameters that set the noise degeneration level. The transient response yields information on loop response time, and thus the switching time needed for frequency agility.

GENERAL DESCRIPTION

Figure 1 is a block diagram of a voltage-controlled oscillator (VCO) with a time-delay feedback loop containing an interferometer and a video amplifier. The interferometer's output is proportional to the phase difference between the direct and delayed outputs from the VCO. If the VCO output is given by $\cos(\omega t)$, the interferometer output after the high-frequency component is filtered out, is given by

$$e(t) = \sin\omega t.$$

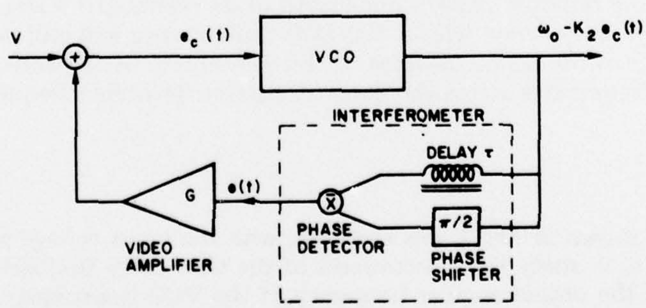

Fig. 1 — Voltage-controlled oscillator with time-delay feedback

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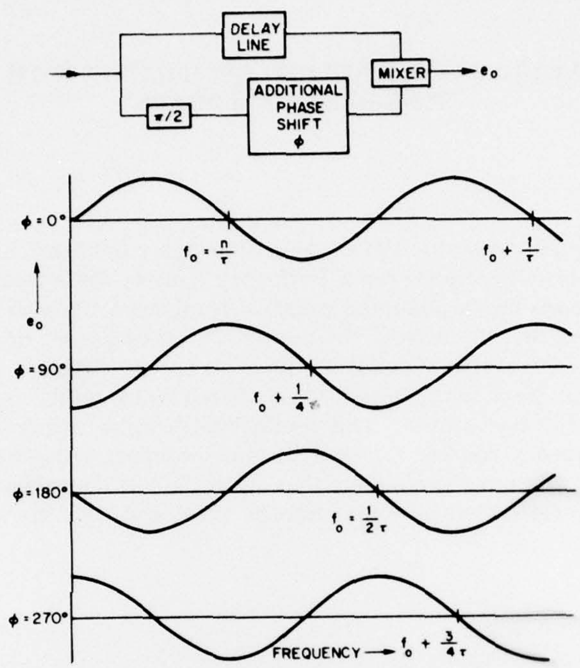

Fig. 2 — Interferometer response with additional phase shift

Therefore, interferometer output voltage vs frequency is a sine wave, so responses repeat, spaced by a frequency equal to the inverse of the delay time, as shown in Fig. 2. For a delay time of 25 ns, the periodicity equals 40 MHz. The whole response curve can be shifted by adding additional phase shift in the direct path [1]. A 90-degree phase shift will shift the whole response pattern one-fourth of its period (10 MHz). In this manner, the interferometer can operate with a ±20-MHz pull-in range and still produce a response every 10 MHz across the band. Because of this periodicity of responses, stable operation occurs at many frequencies across the band of interest, providing frequency agility.

ANALYSIS

The system shown in Fig. 1 was analyzed, with the input voltage as a step function of an amplitude v, to study the effectiveness of the time delay feedback. When input voltage v is zero, the output angular frequency of the VCO is assumed to be ω_o and is related to time delay τ as $\omega_o = 2\pi N/\tau$, where N is an integer. When this relation holds, ω_o becomes one of the stable frequencies, and the output voltage of the interferometer will be zero. Now, if input voltage v is applied, the output frequency of the VCO will change. That change in frequency is given by

$$\Delta\omega = -K_2\, e_c(t) \tag{1}$$

where $e_c(t)$ is the voltage at the VCO input and K_2 is the modulation sensitivity in radians per volt.

Let $\phi(t)$ represent the phase shift due to this frequency change during a time t. The quantity $\phi(t)$ is given by

$$\phi(t) = -\int_o^t K_2\, e_c(t)\, dt. \tag{2}$$

Then, interferometer output voltage $e(t)$, after the high-frequency component is filtered out, is

$$e(t) = K_1 \sin[\phi(t) - \phi(t - \tau)], \tag{3}$$

where K_1 is the phase-detector sensitivity constant in volts per radian.

For $[\phi(t) - \phi(t - \tau)] < 1$, which is satisfied near the stable frequency of operation, one can approximate:

$$e(t) \approx K_1 [\phi(t) - \phi(t - \tau)]. \tag{4}$$

Taking Laplace transforms of Eqs. (1), (2), and (4) yields

$$\Delta\Omega(s) = -K_2\, E_c(s) \tag{5}$$

$$\Phi(s) = -K_2\, E_c(s)/s \tag{6}$$

$$E(s) = K_1 [\Phi(s) - \Phi(s)e^{-\tau s}]. \tag{7}$$

In addition, From Fig. 1 we have

$$E_c(s) = \frac{v}{s} + E(s)\, G(s), \tag{8}$$

where $G(s)$ is the transfer function of the video amplifier.

Using Eqs. (5) to (8), we can show that

$$\Delta\Omega(s) = \frac{-K_2 v}{K_1 K_2 G(s)[1-e^{-\tau s}] + s}. \tag{9}$$

The amplifier in the feedback loop is assumed to be a combination video amplifier and low-pass filter with combined transfer function

$$G(s) = \frac{\alpha\, G_o}{s + \alpha} \tag{10}$$

where G_o is the low-frequency gain of the amplifier and α is the 3-dB cutoff point.

Substituting Eq. (10) in Eq. (9) gives the normalized response

$$F(s) = \frac{\Delta\Omega(s)}{-K_2 v} = \frac{s + \alpha}{s^2 + s\alpha + A(1 - e^{-\tau s})} \tag{11}$$

where $A = K_1 K_2 G_o \alpha$.

Steady State Solution

A steady state solution can be obtained easily from Eq. (11), as

$$\frac{\Delta\omega(\infty)}{-K_2 v} = \lim_{t \to \infty} f(t) = \lim_{s \to o} sF(s) = \frac{1}{K_1 K_2 G_o \tau + 1}. \tag{12}$$

This steady state solution can be used [2] to find the amount of VCO noise degeneration possible with a time-delay feedback system. Let Δf_{osc} represent the FM noise of VCO without feedback. The corresponding equivalent noise voltage at the VCO input is $2\pi\Delta f_{osc}/(-K_2)$. Let Δf be the VCO FM noise with feedback. Then, substituting $v = 2\pi\Delta f_{osc}/(-K_2)$ and $\Delta\omega(\infty) = 2\pi\Delta f$ in Eq. (12), we obtain

$$\Delta f = \frac{\Delta f_{osc}}{K_1 K_2 G_o \tau + 1} . \tag{13}$$

From Eq. (13) it may be noted that the VCO noise is degenerated by a factor of $(K_1 K_2 G_o \tau + 1)$. Therefore, to reduce VCO noise, it is necessary that

$$(K_1 K_2 G_o \tau + 1) \gg 1. \tag{14}$$

Stability Condition

The condition for stability of the feedback system is discussed in Appendix A and is given by

$$K_1 K_2 G_o \alpha \tau^2 < \pi^2/2, \tag{15}$$

which is identical to Eq. (A11).

For the system to be stable and produce noise degeneration, K_1, K_2, G_o, α, and τ should be chosen such that the conditions in both Eqs. (14) and (15) are satisfied.

Transient Response

Formally, the inverse Laplace transform of Eq. (11) gives the transient response. However, the inverse transform is not readily available. The following procedure [3] is used to obtain the transient response. Repeating Eq. (11), we have

$$F(s) = \frac{s + \alpha}{s^2 + s\alpha + A - Ae^{-\tau s}}. \tag{16}$$

Let $z = s^2 + s\alpha + A$. Then

$$F(s) = \frac{s + \alpha}{z} \frac{1}{1 - \frac{A}{z} e^{-\tau s}}. \tag{17}$$

If the expansion of the type

$$\frac{1}{1 - x} = 1 + x + x^2 + \ldots,$$

is used, Eq. (17) becomes

$$F(s) = \frac{s + \alpha}{z} \left(1 + \frac{A}{z} e^{-\tau s} + \frac{A^2}{z^2} e^{-2\tau s} + \ldots\right). \tag{18}$$

Taking the inverse Laplace transform of Eq. (18) yields

$$f(t) = f_1(t) + Au(t - \tau)f_2(t - \tau) + A^2u(t - 2\tau)f_3(t - 2\tau) + \ldots \tag{19}$$

where

$$f_n(t) = L^{-1}\left[\frac{s + \alpha}{(s^2 + s\alpha + A)^n}\right] \tag{20}$$

and $u(t)$ is a unit step function.

The transient response given in Eq. (19) appears to contain an infinite number of terms. However, for any given finite time t, only a finite number of terms are nonzero. Also, each individual term has a physical significance related to the time-delay feedback. In the time range $0 \leqslant t < \tau$, only the first term is nonzero and the rest of the terms are zero, so that the first term gives the system response before the VCO output is applied to the phase detector through the delayed path. In the range $\tau \leqslant t < 2\tau$, the signal through the delayed path is applied to the phase detector, and the transient response is represented by the first two terms, since the remaining terms are still zero. In general, for range $(M - 1)\tau \leqslant t < M\tau$, the first M terms give the transient response because the higher order terms are still zero.

The transient response is not yet complete, because the inverse transform shown in Eq. (20) is not readily available. A closed form solution for $f_n(t)$ is obtained by using the following procedure. First, consider the inverse Laplace transform,

$$S_n(t) = L^{-1}\left[\frac{1}{(s^2 + s\alpha + A)^n}\right].$$

This can be written as

$$S_n(t) = L^{-1}\left[\frac{1}{(s+a)^n\,(s+b)^n}\right] \tag{21}$$

where $a = \frac{\alpha}{2} - j\omega_1$, $b = \frac{\alpha}{2} + j\omega_1$, and $\omega_1 = \sqrt{A - \frac{\alpha^2}{4}}$. It will be shown later that $\alpha^2/4 \ll A$. Therefore, ω_1 is real.

The inverse Laplace transform shown in Eq. (21) is readily available [4]. It is given by

$$S_n(t) = \frac{\sqrt{\pi}}{(n-1)!}\left(\frac{t}{a-b}\right)^{n-½} e^{-\frac{(a+b)t}{2}}\, I_{n-½}\left(\frac{a+b}{2}\right)t$$

where $I_{n-½}$ is a modified Bessel function of the first kind. Substituting for a and b from Eq. (21), we obtain

$$S_n(t) = \frac{\sqrt{\pi}}{(n-1)!}\left(\frac{t}{2\omega_1}\right)^{n-½} e^{-\frac{\alpha t}{2}}\, J_{n-½}(\omega_1 t) \tag{22}$$

where $J_{n-½}$ is the Bessel function of the first kind.

From Eqs. (20) and (21) it may be noted that $f_n(t)$, which can be obtained if $s_n(t)$ is known, is given by

$$f_n(t) = s_n(t) - s_n(+0) + \alpha\, s_n(t) \tag{23}$$

where $s_n(t)$ is the first derivative of $s_n(t)$.

By noting $s_n(+0) = 0$ and substituting $s_n(t)$ from Eqs. (22) and (23) and simplifying, we can show that

$$f_n(t) = \frac{\sqrt{\pi}}{(n-1)!}\left(\frac{t}{2\omega_1}\right)^{n-½} e^{-\frac{\alpha t}{2}}\left[\omega_1 J_{n-3/2}(\omega_1 t) + \frac{\alpha}{2} J_{n-1/2}(\omega_1 t)\right]. \tag{24}$$

In terms of spherical Bessel functions j_n, one can show [5] that

$$f_n(t) = \frac{1}{(n-1)!}\left(\frac{t}{2\omega_1}\right)^{n-1}(\omega_1 t)\, e^{-\frac{\alpha t}{2}}\left[j_{n-2}(\omega_1 t) + \frac{\alpha}{2\omega_1} j_{n-1}(\omega_1 t)\right] \tag{25}$$

Hence, the transient response is given by Eq. (19), with $f_n(t)$ as shown in Eq. (25).

NUMERICAL RESULTS

With Eqs. (19) and (25), the transient response is computed for typical values of the system parameters, K_1 = 0.2 V/rad, K_2 = 3.5 × 10^8 rad/sV, α = 6.283 × 10^3 rad/s (corresponds to an amplifier filter bandwidth of 1 kHz), and τ = 25 ns. Figure 3 shows the normalized transient response with the above parameters and for the amplifier gains

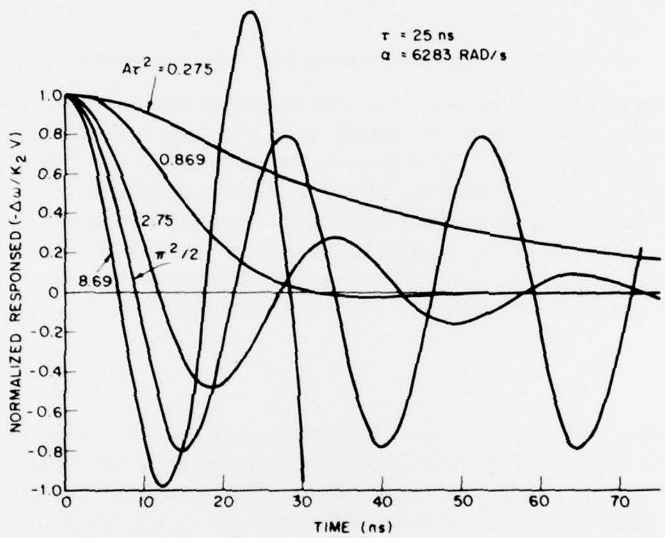

Fig. 3 — Transient response

of 60 dB (G_o = 10^3), 70 dB (G_o = 3.162 × 10^3), 80 dB (G_o = 10^4), 85.0823 dB (G_o = 1.7952 × 10^4), and 90 dB (G_o = 3.162 × 10^4). The corresponding values of $A\tau^2$ (= $K_1K_2G_o\alpha\tau^2$) are 0.275, 0.869, 2.75, $\pi^2/2$, and 8.69. For the system to be stable, $A\tau^2$ must be less than $\pi^2/2$ (= 4.9348). From Fig. 3, it is evident that for $A\tau^2 >$ 4.9348 the system transient response is oscillatory with increasing amplitude with time. This indicates that the system is unstable. For $A\tau^2 = \pi^2/2$, the response is oscillatory with approximately constant amplitude, indicating that the stability condition given by Eq. (15) is highly accurate. For $A\tau^2$ = 2.75, the transient response is oscillatory, but the oscillations are damped with time, suggesting that the system is stable. As the amplifier gain is reduced, making $A\tau^2$ smaller, the damped oscillations almost vanish for $A\tau^2$ = 0.869 (which may be called the critically damped case). For smaller values of $A\tau^2$, the transient response decreases with time (similar to an exponentially decreasing curve) and takes longer to reach steady state.

These results indicate that a value for $A\tau^2 \approx 1$ may be a best compromise for faster loop response and acceptable damped oscillations. For the critically damped case, the loop response reaches steady state in a time span of about 8τ. For a given value of $A\tau^2$, the loop response time is smaller for smaller values of τ, as shown in Fig. 4. However,

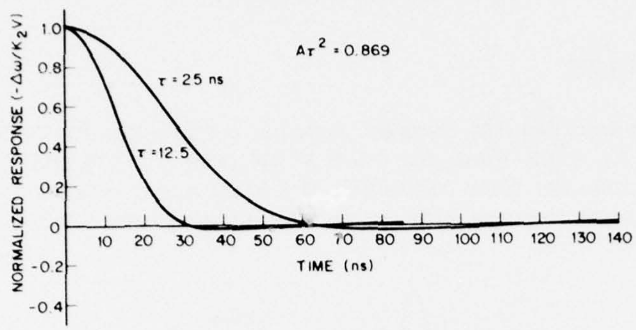

Fig. 4 — Frequency response with τ as a parameter

if the time is normalized with respect to τ, the transient responses for both cases are approximately similar, except that the steady state response is smaller for smaller τ. Since $A = K_1 K_2 G_o \alpha$, changing K_1 and K_2 will have the same effect as changing G_o. However, changing α has approximately the same effect on the transient response as changing G_o, but the steady state response is independent of α. For these reasons, α should be chosen as small as possible, and $K_1 K_2 G_o$ should be chosen as large as possible, for a given τ such that $A\tau^2 = 1$ and $K_1 K_2 G_o \tau \gg 1$, which will satisfy the requirements of noise degeneration and system stability conditions given by Eqs. (14) and (15).

CONCLUSIONS

A voltage-controlled oscillator with time-delay feedback has been analyzed. By properly choosing system parameters, low noise from the degeneration and frequency agility from the periodic response can be obtained in one source. The conditions for system stability and the amount of noise degeneration are expressed in terms of system parameters. Effects of different parameters on system stability and noise degeneration are discussed in detail. It is shown that the open-loop gain should be chosen as large as possible for good noise degeneration, and the bandwidth should be adjusted to meet the stability condition.

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Appendix A

STABILITY CRITERION

The Nyquist stability criterion is used to find the condition under which the feedback system is stable and to identify the system parameters on which this condition depends. This criterion furnishes a graphical method for determining the stability of a system. The system is stable if a plot of its open-loop transfer function $Z(s)$ for a succession of values of s, encircling the entire right half of the s-plane in the clockwise direction, makes a number of counterclockwise revolutions about the critical point $(-1 + j0)$ equal to the number of the poles of $Z(s)$ in the right half of the s-plane.

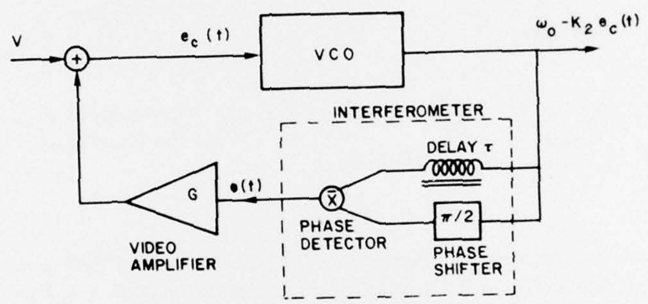

Fig. A1 — Voltage-controlled oscillator with time-delay feedback

For the system in Fig. A1, the open-loop transfer function is given by

$$Z(s) = \frac{A(1-e^{-\tau s})}{s(s + \alpha)} \tag{A1}$$

From Eq. (A1), note that there are no poles in the right half of the s-plane. The only pole at $S = -\alpha$ is in the left half. Also, there are no poles on the imaginary axis. Since $Z(s)$ becomes zero for $s \to \infty$, to apply the Nyquist criterion one needs a polar plot of $Z(s)$ with $s = j\omega$ and for the range of values $-\infty \leqslant \omega \leqslant \infty$ only. As can be noted, a polar plot for the frequency range $-\infty$ to 0 is the mirror image about the horizontal axis of the plot for the frequency range 0 to ∞. Thus, to evaluate the polar plot of a transfer function, the only frequency range to be considered is from 0 to ∞. By substituting $s = j\omega$ in Eq. (A1) and finding real part $Z_R(j\omega)$ and imaginary part $Z_I(j\omega)$ of transfer function $Z(j\omega)$, it can be shown that

$$Z_R(j\omega) = \frac{A}{\alpha^2 + \omega^2}\left(\alpha\tau \frac{\sin\omega\tau}{\omega\tau} - 2\sin^2\frac{\omega\tau}{2}\right) \tag{A2}$$

$$Z_I(j\omega) = \frac{-A}{\alpha^2 + \omega^2}\left[\sin\omega\tau + \alpha\tau\frac{\sin(\omega\tau/2)\,\sin(\omega\tau/2)}{(\omega\tau/2)}\right]. \tag{A3}$$

From Eqs. (A2) and (A3), note that there is no simple way of sketching the polar plot if parameters A, α, and τ are not specified. Fortunately, for the system under consideration the range of parameters is such that a simplification is possible. For $\alpha\tau$ of the order of 10^{-4}, the first term in the parentheses on the right hand side of Eq. (A2) can be neglected, except possibly when $\omega\tau \approx 2N\pi$. (In that case, both terms will be close to zero and $Z_R(j\omega)$ will be very small.) Similarly, the second term in the brackets of Eq. (A3) can be neglected, except possibly at or very near $\omega\tau = N\pi$. In that case, both terms will be very small, which in turn makes Z_I small. These approximations and observations are used only to simplify the procedure in obtaining the polar plot. No approximation is needed in obtaining the exact condition for stability, as will be shown later. Keeping aside the term $A/(\alpha^2 + \omega^2)$, which is common for both Z_R and Z_I, it may be noted that Z_R varies as $-2\sin^2(\omega\tau/2)$ and Z_I varies as $-\sin(\omega\tau)$. These two functions are sketched in Fig. A2 as a function of $\omega\tau$.

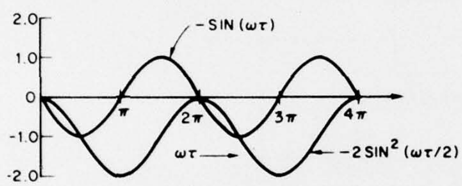

Fig. A2 — Variation of Z_R and Z_I, except for the factor $A/(\alpha^2 + \omega^2)$

In the range of parameters of interest, α is of the order of 10^4 and τ is of the order of 10^{-8}. Therefore, for $\omega\tau > \pi/2$, $\omega \gg \alpha$ and the factor $A/(\alpha^2 + \omega^2) \approx A/\omega^2$. Using these observations and Fig. A2, one can sketch the polar plot of $Z(j\omega)$, which is shown in Fig. A3 for the range $0 \leqslant \omega < \infty$. For the range $-\infty < \omega \leqslant 0$, the polar diagram will be the mirror image about the Z_R axis, and it is not shown. From the sketch in Fig. A3

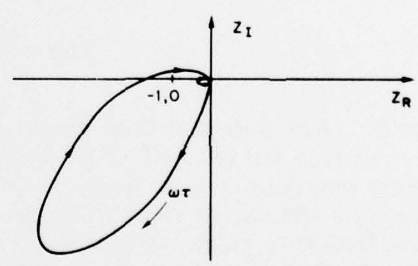

Fig. A3 — Sketch of a polar plot for $Z(j\omega)$, for $A > A_c$

it is clear that if the real part $Z_R < -1$ $\left(|Z_R| \geqslant 1\right)$ when $Z_I = 0$, the critical point is encircled counterclockwise by the polar plot. Therefore, the system will be unstable. Since the magnitude of Z_R depends on A, for the system to be stable A should be smaller than that for which $Z_R = -1$ and $Z_I = 0$.

Next, it is possible to find the value of A corresponding to $Z_R = -1$ and $Z_I = 0$. This will be done using Eqs. (A2) and (A3), first without making any approximations. Equating $Z_I = 0$, we obtain from Eq. (A3)

$$\sin\omega\tau = -2\frac{\alpha}{\omega}\sin^2\frac{\omega\tau}{2}. \tag{A4}$$

Equating $Z_R = -1$, we obtain from Eq. (A2)

$$\frac{A}{\alpha^2 + \omega^2}\left(\frac{\alpha}{\omega}\sin\omega\tau - 2\sin^2\frac{\omega\tau}{2}\right) = -1. \tag{A5}$$

Substituting from Eq. (A4) for $\sin\omega\tau$ in Eq. (A5) yields

$$2A\sin^2\frac{\omega\tau}{2} = \omega^2. \tag{A6}$$

From Eqs. (A4) and (A6), we obtain

$$\cos\frac{\omega\tau}{2} = -\frac{\alpha}{\sqrt{2A}} \tag{A7}$$

or

$$\omega_c = \omega = \frac{2}{\tau}\cos^{-1}\left(-\alpha/\sqrt{2A}\right) \tag{A8}$$

where ω_c is the frequency at which $Z_R = -1$ and $Z_I = 0$, for given values of τ, α, and A. Substituting ω_c from Eq. (A8) for ω in Eqs. (A6) and (A7), we obtain

$$\sqrt{2A - \alpha^2} = \omega_c = \frac{2}{\tau}\cos^{-1}\left(-\alpha/\sqrt{2A}\right). \tag{A9}$$

Equation (A9) is the exact stability condition from which can be found the value of $A = A_c$ that satisfies Eq. (A9) for given values of α and τ. The system will then be stable if the parameters are chosen such that $A < A_c$. However, Eq. (A9) is a transcedental equation that is difficult to solve and does not provide convenient interpretation. Fortunately, for the range of system parameters in which we are interested, $\alpha^2 \ll 2A$, which allows us to neglect α^2 in Eq. (A9) and approximate $\cos^{-1}\left(-\alpha/\sqrt{2A}\right) \approx \pi/2$. Then, from Eq. (A9) we have

$$A_c \approx \pi^2/2\tau^2 \tag{A10}$$

and

$$\omega_c \approx \pi/\tau.$$

Therefore, for the system to be stable, the system parameters should be chosen such that

$$A\tau^2 < \pi^2/2. \tag{A11}$$

The numerical results presented in this report validate the accuracy of the approximations made in arriving at Eq. (A11).